

# Experimental Comparison of Bayesian Outdoor Vehicle Localization Filters

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**Abstract**—Localizing a vehicle consists in estimating its state by merging data from proprioceptive sensors (inertial measurement unit, gyrometer, odometer, etc.) and exteroceptive sensors (GPS sensor). A well known solution in state estimation is provided by the Kalman filter. But, due to the presence of nonlinearities, the Kalman estimator is applicable only through some alternatives among which the Extended Kalman filter (EKF), the Unscented Kalman Filter (UKF) and the Divided Differences of 1<sup>st</sup> and 2<sup>nd</sup> order (DD1 and DD2). We have compared these filters using the same experimental data. The results obtained aim to rank these approaches by their performances in terms of accuracy, confidence and consistency.

## I. INTRODUCTION

In the intelligent vehicle applications, the extended Kalman filter (EKF) has unquestionably been up to now, the dominating state estimation technique [1], [2]. More recently, some new methods have been developed in order to improve the nonlinear estimation. They include some other variants of the Kalman filter such as the Divided Differences of first and second order (DD1 & DD2) [3], [4] and the unscented Kalman filtering (UKF) [5]. The principle of these new approaches is based on the linearization of the process and measurement functions by statistical linear regression functions, through sampling points in the region around the state estimatee.

Up to now there is not much work related to the comparison of performances of the more recent filters. A study carried out in [6] aims to evaluate some Kalman filter variants (EKF, DD1, DD2, UKF) capacities to linearize the process and measurement models. This work theoretically compares the filters' performance separately for the prediction step and the correction step, but does not analyze their overall performances. [7] shows the performances of these estimators (EKF, DD1, DD2 and UKF) in their predictive steps in road vehicles localization. The objective of this paper is to complement the work already carried out in [7] by taking into account the overall localization process (both the predictive and corrective steps). In order to ensure the comparison be-

tween these methods, two criteria of performance are set and applied. The first criterion concerns the evaluation of the accuracy of each method. We use a reference trajectory provided by a centimetric RTK GPS. The next studied criteria relate to the size of the  $2\sigma$  scaled confidence envelopes, the  $2\sigma$  uncertainty ellipse areas and the Normalized Innovation Squared (NIS), from which each filter consistency will be deduced. This paper is organized as follows. Section 2 provides an overview of the Kalman Filter approaches (EKF, UKF, DD1, DD2) for nonlinear estimation. Section 3 presents the system modeling and the comparison criteria that we use in this work. In section 4, we describe the experimental environment and analyse the results of the 4 filters.

## II. REVIEW OF THE KALMAN FILTER APPROACHES FOR NONLINEAR ESTIMATION

### A. The Extended Kalman Filter (EKF)

The main difference between traditional and extended Kalman filter appears in the computation of the various matrices. In the KF, the process and the measurement matrices are composed of "true" linear functions; whereas in the EKF, these matrices (called Jacobian matrices) are composed of Taylor first order linearized functions. Although the EKF has been shown reliable in many practical driving situations, it has some well known drawbacks. A major one concerns the hypothesis related to the point of the linearization. Theoretically, the nonlinear process function  $f$  is linearized around the true current state. But in the implementation, this function is linearized around the estimated value of  $X$ , leading to an additional error [7]. Moreover, another evident limitation of this filter concerns the possibility of computation of the Jacobian matrices. For very complex systems, strong nonlinearities can generate system instability problems; therefore the theoretical calculation of these matrices can simply become impossible, for example when the process or measurement functions are non-differentiable. However for intelligent vehicles applications, most of the used functions are differentiable, so the main drawback

concerns linearization. In order to bypass these limitations, some other methods based on a derivative free approach are presented in the following. These methods are shown often more powerful than the EKF and include UKF [5], DD1 & DD2 [4].

### B. The Unscented Kalman Filter (UKF)

The UKF is an application of the unscented Transformation to a mean square recursive estimation [5]. This transformation is a method for calculating the statistics of a random variable that undergoes a nonlinear process. We consider a random propagating variable  $X$  through a nonlinear function,  $Y = f(X)$ . A set of *sigma points* with mean  $\bar{X}$  (mean of  $X$ ) and covariance  $P_{xx}$  (covariance of  $X$ ), is deterministically chosen. The nonlinear function  $f$  is applied to each point to yield a cloud of transformed points with statistics  $\bar{Y}$  and  $P_{yy}$ . The  $n$ -dimensional random variable  $X$  is approximated by  $2n + 1$  weighted *sigma points*.

This transformation is built following the steps below:

- The sigma points are propagated through the nonlinear function. For  $i = 0, \dots, 2n$

$$\mathcal{Y}_i = f[\mathcal{X}_i] \quad (1)$$

- The mean is calculated as the weighted mean of the transformed points:

$$\bar{Y} = \sum_{i=0}^{2n} W_i \mathcal{Y}_i \quad (2)$$

where  $W_i$  is the weight of the  $i^{\text{th}}$  point.

- The covariance is obtained according to Eq. 3

$$P_{yy} = \sum_{i=0}^{2n} W_i [\mathcal{Y}_i - \bar{Y}][\mathcal{Y}_i - \bar{Y}]^T \quad (3)$$

### C. The Divided Differences Kalman Filter of first and second Order (DD1 & DD2)

These filters' formulations were proposed by Norgaard et al [4]. Both are based on the Stirling interpolation (presented in the following), and their implementation methods are very similar. In the DD1, we are limited to the 1<sup>st</sup> order interpolation, and in the DD2 the functions are linearized at the 2<sup>nd</sup> order. The DD1 & DD2 filters differ from the EKF in the fact that the Jacobian matrices are replaced by divided differences. Therefore the correction steps are the same. The main difference appears in the filters' covariance matrices update.

In the EKF we used to linearize the function from a Taylor series development of first order. In order to implement the Sterling interpolation two operators are defined. Let us consider a one dimensional interval with length  $\xi$ :

$$\delta f(X) = f\left(X + \frac{\xi}{2}\right) - f\left(X - \frac{\xi}{2}\right) \quad (4)$$

$$\mu f(X) = \frac{1}{2}(f\left(X + \frac{\xi}{2}\right) + f\left(X - \frac{\xi}{2}\right)) \quad (5)$$

The Stirling interpolation formula is obtained by applying both the above operators to the mean value  $\mu_X$  rather than directly to  $X$ . The second order interpolation gives

$$f(X) \simeq f(\mu_X) + f'_{DD}(\mu_X)(X - \mu_X) + \frac{f''_{DD}(\mu_X)}{2!}(X - \mu_X)^2 \quad (6)$$

where  $\mu_X$  is the mean value of distribution  $X$  and

$$f'_{DD}(\mu_X) = \frac{f(\mu_X + \xi) - f(\mu_X - \xi)}{2\xi} \quad (7)$$

$$f''_{DD}(\mu_X) = \frac{f(\mu_X + \xi) + f(\mu_X - \xi) - 2f(\mu_X)}{\xi^2} \quad (8)$$

During the DD1 and DD2 filters implementation, a Householder triangulation is introduced, in order to compute the characteristic divided difference matrices. These matrices are then used in order to get the process and measurement noise covariance matrices, as well as the predicted and corrected state error covariance matrices. More details can be found in [4], [6], [7].

## III. SYSTEM MODELING AND COMPARISON CRITERIA

### A. Vehicle model and GPS model

All the filters that are compared are based on the kinematic model presented in equation 9.

$$\begin{cases} x_{k|k-1} &= x_{k-1|k-1} + \\ &V_k T \cos(\psi_{k-1|k-1} + T\dot{\psi}_k/2) \cos(\delta_k) \\ y_{k|k-1} &= y_{k-1|k-1} + \\ &V_k T \sin(\psi_{k-1|k-1} + T\dot{\psi}_k/2) \cos(\delta_k) \\ \psi_{k|k-1} &= \psi_{k-1|k-1} + \dot{\psi}_k T \end{cases} \quad (9)$$

where  $T$  is the time period,  $[x_k, y_k]^T$  is the position vector,  $V_k$  is the vehicle velocity and  $\delta_k$  is the steering front wheel angle. This angle is used to take into account the kinematic constraints on the vehicle [8].  $\psi_k$  and  $\dot{\psi}_k$  represent the yaw angle (heading) and the yaw rate, respectively.

The GPS observation model is linear and is the same for all filters, with the corresponding observation matrix

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (10)$$

### B. Comparison criteria

In order to evaluate and then compare the presented filter performances, many measures can be used. In this work, we focused on the accuracy measure, the confidence on the outputs and then the consistency of various filters.

- **Accuracy:** For our localization application it is more appropriate to use as accuracy measure, the Average Euclidean Error **AEE** [9] defined as

$$AEE(\hat{X}) = \frac{1}{M} \sum_{i=1}^M \sqrt{\hat{X}_i^T \hat{X}_i} \quad (11)$$

Where  $M$  is a number of trials,  $\tilde{X} = X - \hat{X}$  is the estimation error,  $X$  is the real state vector and  $\hat{X}$  is its estimate. AEE is chosen because it better approximates the true mean error, that means the true average Euclidean distance between the real value and the estimate.

- **Confidence in the output:**

- Firstly, we focus on the  $2\sigma$  confidence envelopes in order to evaluate the level of confidence we can have on each filter's output. Evaluating the various filters uncertainties can derive in the consistency of each filter's estimated error assessment.
- Secondly, the  $2\sigma$  uncertainty ellipse areas are considered. In fact, the results of the filters are given *a posteriori* with an uncertainty symbolized by an ellipse. In order to obtain the size of the axes of these ellipses, it is necessary to compute the eigenvalues of the covariance matrix  $P$ . These values are weighted with a factor  $k = \sqrt{-2\log(1 - P_a)}$ , where  $P_a$  is the membership probability [10].

- **Consistency:** Various filters' consistency is studied. A state estimatee  $\hat{X}$  with covariance matrix  $P$  is called consistent if it satisfies equation 12 [6], [11].

$$E \left[ (X - \hat{X}) (X - \hat{X})^T \right] \equiv E \left[ \tilde{X} \tilde{X}^T \right] \leq P \quad (12)$$

The Normalized Innovation Squared (NIS) measure,  $\epsilon$ , is used [11] to characterize the filters' consistency.

$$\epsilon = \tilde{z}^T S^{-1} \tilde{z} \quad (13)$$

$\tilde{z}$  is the filter innovation and  $S$  the innovation covariance matrix. Under the hypothesis that the filters are consistent and approximately linear-Gaussian,  $\epsilon$  is  $\chi^2$  (chi-square) distributed with  $\dim(Z)$  degrees of freedom. The average value of the NIS  $\epsilon$  then tends toward the dimension of the observation vector  $Z = [x_{GPS}, y_{GPS}]$

$$E[\epsilon] = n, \quad \text{with } n = 2 \quad (14)$$

Therefore, a filter will be called consistent, if the average NIS is less or equal to 2.

#### IV. EXPERIMENTAL RESULTS

##### A. Test track and collected data

The tests carried out in this work used real data collected with It is assumed that the slope and bank angles remain negligible. The experimental data used in equation 9 were directly provided by an inertial measurement unit Crossbow VG400 (yaw rate  $\dot{\psi}$ ), an odometer fixed to the front axis (vehicle speed  $V$ ) and a coder that recorded the steering angle at the front wheel ( $\delta$ ). A low cost GPS directly provided correction data and initial states. The state noise was derived from various sensor noises. In our experiment, we have  $\sigma_{odometer} = 0.005m/s$

and  $\sigma_{gyro} = 0.05rad/s$ . The complete track has a length of approximately  $5.5km$ . During the tests, the reference trajectory was obtained by using a fusion of a very accurate INS and a RTK<sup>1</sup> GPS (Thales).

During the first scenario (complete test track, see figure

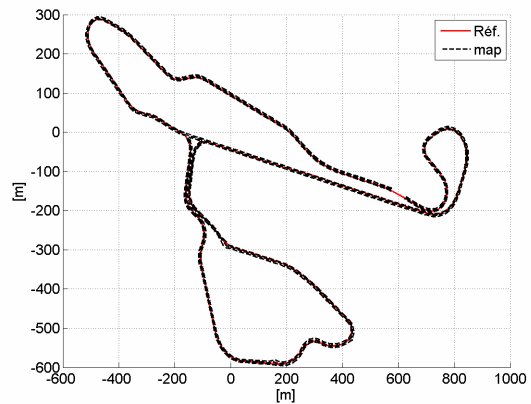


Fig. 1. Test track in scenario 1

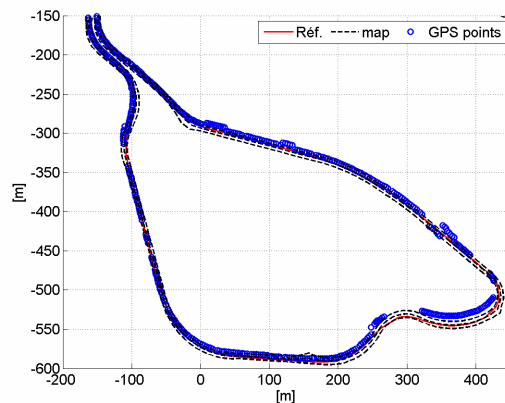


Fig. 2. Zoom on Forest Area in scenario 1

1), all the sensors were synchronized thanks to GPS timestamps, and the system provided an output at the frequency of  $5Hz$ . In the area surrounded by trees (see figure 2 around  $(400, -500)$ ) the GPS points have an error of more than  $10m$  from the reference.

The second scenario took place on a part of the road track described in figure 3. During this scenario, the GPS sensor was switched off from time to time. During such periods, the localization was performed with only proprioceptive sensors and the filters ran in the predictive step. The update was performed only in presence of GPS.

##### B. Accuracy

1) *Scenario 1:* Figure 4 presents the positioning Euclidean errors for the 4 presented methods. The methods show rather comparable errors. Table I presents the mean

<sup>1</sup>Real Time Kinematic

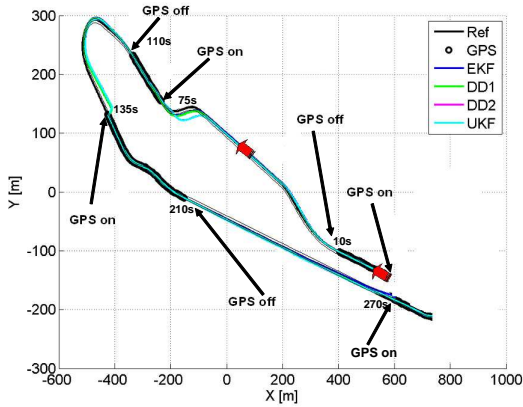


Fig. 3. Filters localization in scenario 2

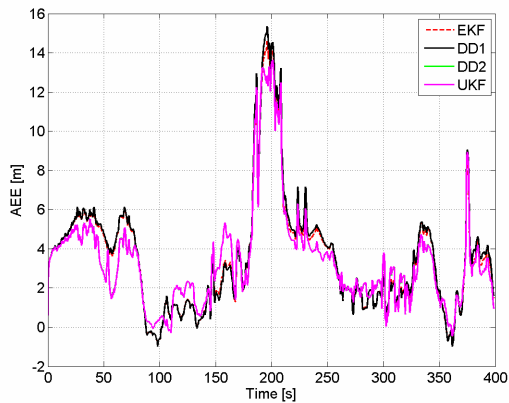


Fig. 4. Euclidean positioning error in scenario 1

and maximum values of the Euclidean errors. The analysis of this table brings a confirmation of what figure 4 indicates. The performances of the EKF, UKF, DD1 and DD2 filters are globally similar (2.91% difference). When we focus on these results, we can notice that EKF and DD1 have the same mean error  $3.093m$ , they appear globally more accurate than UKF and DD2 (mean AEE  $3.186m$  and  $3.185m$ ).

Nevertheless the max values of the error, which are reached in the area in figure 2 (from  $125s$  to  $220s$ ), reveal that the UKF and DD2 have a better behavior. This result contrasts with the global performance order established from mean AEE, the reason for this is in the following. In fact, this area is characterized by very poor quality GPS data combined with strong nonlinearities

TABLE I  
MEAN AND MAX AEE IN SCENARIO 1

Method	Mean AEE (m)	Max. AEE (m)
EKF	3.093	15.274
UKF	3.186	14.732
DD1	3.093	15.273
DD2	3.185	14.732

in the prediction (strong turns in the trajectory). In such situations filter outputs are more influenced by the predictive step. UKF and DD2 better handle such nonlinearities and are less influenced by updating GPS data. Considering Table I, the second order filters (UKF and DD2) are slightly more accurate (respective max AEE  $14.732m$ ) than first order filters (EKF max AEE  $15.274m$ , DD1 max AEE  $15.273m$ ) in such a situation. But in each group the filters behave very similarly. To conclude, it appears that following given situations, EKF and DD1 have better performances than UKF and DD2 and vice versa. But globally, first order filters are recommended for such a scenario.

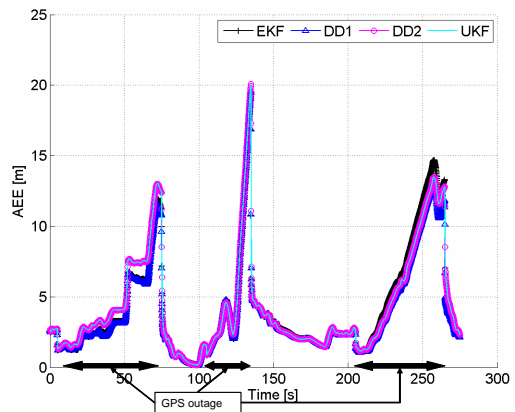


Fig. 5. Euclidean positioning Error in scenario 2

2) *scenario 2*: The Euclidean positioning error in this scenario is shown in figure 5. First of all, it is remarkable that in presence of GPS signal, all the methods show rather comparable errors (like in scenario 1). Important differences appear when GPS is switched off. This is an interesting illustration of the theory : in fact, these filters are theoretically different in the manner they handle the nonlinear process fonction. These theoretical differences are presented in [6] and [7].

In this scenario, the common GPS observation model is linear. Thus, the correction stage will not show important differences between filters. However, during the GPS outages, figure 5 confirms that EKF and DD1 (first order filters) react similarly and are more accurate than UKF and DD2, which are also similar.

During the first GPS outage, EKF and DD1 mean AEE ( $4.89m$  and  $4.73$ ) is almost  $1m$  lower than DD2 and UKF mean AEE ( $5.73m$  and  $5.75m$ ). The maximum AEE has the same order. During the second GPS outage, the AEE differences are reduced to about  $15cm$  for the mean values and  $50cm$  for the max. But the first order filters are still more accurate. Figure 6 presents the X-axis and Y-axis positioning error. It reveals that the difference on the X-axis is more important than on the Y-axis. The explanation is found in the modeling error and the cumulated error of second order filters DD2 and UKF. In presence of trajectory non linearity, these filters react as

TABLE II  
MEAN AND MAX AEE DURING GPS OUTAGES (SCENARIO 2)

Method	GPS outage 1 [10 – 75s]		GPS outage 2 [110 – 135s]		GPS outage 3 [210 – 270s]	
	meanAEE(m)	maxAEE(m)	meanAEE(m)	maxAEE(m)	meanAEE(m)	maxAEE(m)
EKF	4.89	11.95	4.87	19.53	7.58	14.50
DD1	4.73	11.56	4.88	19.59	6.68	13.23
DD2	5.73	13.05	5.02	20.08	6.96	13.48
UKF	5.75	13.00	5.02	20.08	6.93	13.48

if the vehicle turns earlier than it should be (see figure 3). In fact, the bias between the first order filters and the second order filters comes from the way the estimatee is computed. Using velocity and heading, the primary computation is done in the polar space and the result is returned in the Cartesian space. This polar to Cartesian conversion problem was handled in [7], and it derives in a cumulative error for second order filters, which is visible on figure 3.

During the third GPS outage, the filters still react similarly with very close errors, except for EKF which is 1m worse. Examining figure 6, we can see that all the filters deviate progressively on the Y-axis (from 210s). However, EKF deviates more than the others. Considering that the road configuration is linear here, this deviation comes from the sensors noise modeling (gyro) and the inadequate initialization before the GPS was switched off. The results given in table II reveal that, considering the defects cited above (sensors noise and inadequate initialization), DD1 is more accurate and shows a better robustness than EKF and second order filters in such situations.

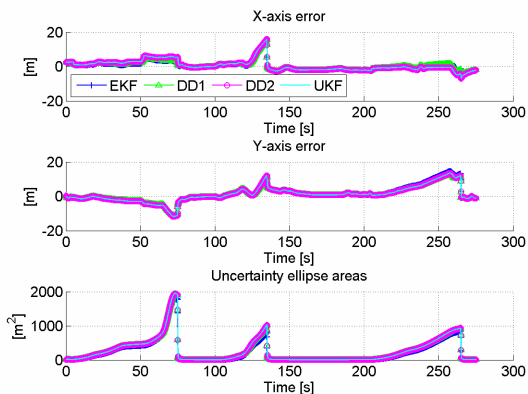


Fig. 6. X-axis (top) Y-axis (middle) positioning error and  $2\sigma$  uncertainty ellipse areas (bottom)

### C. Consistency

1) *Scenario 1*: In order to study the confidence in those estimatee, we compute the  $2\sigma$  scaled envelopes provided by each filter (95% probability region). On figures 7 and 8 we superposed the positioning axis error

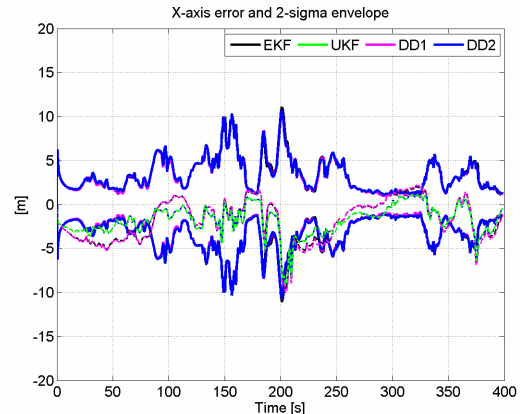


Fig. 7. X-axis  $2\sigma$  envelope and error in scenario 1

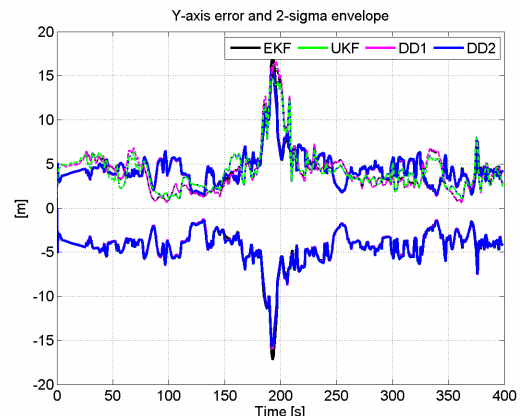


Fig. 8. Y-axis  $2\sigma$  envelope and error in scenario 1

and the associated  $2\sigma$  envelope. For all the filters, we can see that the estimation error on each axis is most of the time inside the corresponding envelope. This means that globally, the error estimatee given by the filters through their covariance matrices is consistent with the real positioning estimation error. The forest area is an exception. With poor quality GPS data, filter envelopes grow considerably (see figures 7 and 8), deriving in a loss of confidence in the estimation. However all the filterenvelopes are almost identical.

Figure 9 shows the NIS of the position innovation normalized to a 95% probability region, assuming a  $\chi^2$  distribution. Most of the time the normalized NIS of EKF, UKF, DD1 and DD2 are below 2.0. This means that

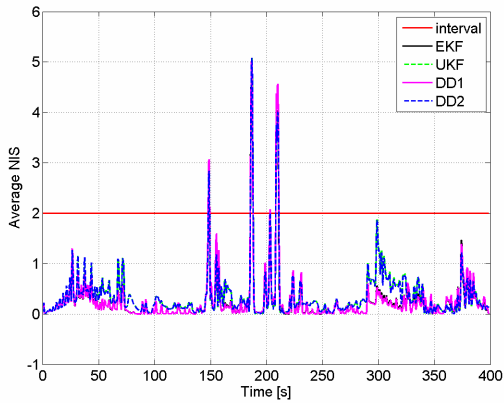


Fig. 9. Filters NIS in scenario 1

the estimated filter uncertainties are most of the time consistent with the true estimation error, considering 95% probability region. Once more an exception appears in the forest area (from 125s to 220s), where NIS values are often above 2.0. This last measure confirms that of the  $2\sigma$  envelopes: the four filters behave similarly, the differences between their errors and their uncertainty are very small.

2) *Scenario 2*: The confidence of the presented filters output is tackled under the  $2\sigma$  uncertainty ellipses analysis. Figure 6 (bottom) shows the filters ellipse areas. It reveals that the uncertainty ellipse areas grow considerably during GPS outages. During the first outage period [10–75s], these areas almost reach to  $2000m^2$ . During the other GPS outage periods the maximal areas are around  $1000m^2$ . The behavior of various filters is very similar: the uncertainty is low when correction data are used, and high if not.

The uncertainty growth is more important during the first GPS outage. This part of the scenario is characterized by multiple trajectory nonlinearities combined with a long period without correction. During the second GPS outage, the trajectory is highly non linear, and the outage lasted 25s. The growth here is comparable to that during the third outage (linear, 60s). These observations reveal that the uncertainty ellipse areas growth is also a function of the system non linearity: the stronger the nonlinearity, the stronger the growth.

## V. CONCLUSION

In this paper an experimental comparative study of 4 Kalman based localization approaches (EKF, UKF, DD1, and DD2 filters) were presented. Previous works [6], [7] exhibit major differences as well as similarities between those filters. [6] shows theoretical differences whereas [7] analyses practical differences during the prediction steps. Nevertheless practical experiments taking into account the whole localization process (both prediction and correction steps) exhibit minor differences. The differences observed during the prediction step in terms

of accuracy and uncertainty (due to the linearization or the use of the sigma points) are strongly reduced during the correction step.

According to our experiments, the choice of a given filter will depend on the situation: this means, the presence and the quality of correction data, or the presence of strong nonlinearities on the trajectory. Therefore

- if there is no GPS signal outage for a long time during the navigation, or if the GPS signal is of good quality (if the correction step runs efficiently), then it is not easy to propose a favorite filter. The use of one or another among the presented filters brings almost insignificant amelioration in terms of accuracy, as well as the confidence of each estimatee.
- if the GPS signal is of poor quality, in presence of strong nonlinearities we should consider UKF or DD2 which seem more robust in such situations.
- if the GPS signal is absent for a long time, the EKF or the DD1 are recommended as mentioned in [7], but our study also reveals that DD1 remains, to our best tunings, the most robust filter among the four.

In further work, these filters will be compared to other estimators which are theoretically assumed more robust, such as multiple model filters or Monte Carlo filters.

## REFERENCES

- [1] I. Abuhadrous, F. Nashashibi, and C. Laugeau, “3-d land vehicle localization: a real-time multi-sensor data fusion approach using rtmmaps,” in *11th International Conference on Advanced Robotics*, (University of Coimbra, Portugal), June 30th-July 3rd 2003.
- [2] J. L. et al., “Multisensorial data fusion for global vehicle and obstacles absolute positioning,” in *IEEE Intelligent Vehicles Symposium*, (Columbus, OH, USA), June 9-11 2003.
- [3] K. Ito and K. Xiong, “Gaussian filters for nonlinear filtering problems,” *In IEEE Transaction on Automatic Control*, vol. 45, May 2000.
- [4] M. Norgaard, N. K. Poulsen, and O. Ravn, “Advances in derivative-free state estimation for nonlinear systems,” revised edition of the technical report imm-rep-1998-15, Technical University of Denmark, 2000.
- [5] S. J. Julier and J. Uhlmann, “A new extension of the kalman filter to nonlinear systems,” in *SPIE Aerosense International Symposium on Aerospace/Defense Sensing, Simulation and Controls*, (Orlando, Florida), September 1997.
- [6] T. Lefebvre, H. Bruyninckx, and J. D. Schutter, “Kalman filters for nonlinear systems: A comparison of performances,” *The International Journal of Control*, vol. 77, May 2004.
- [7] B. Mourllion, D. Gruyer, A. Lambert, and S. Glaser, “Kalman filters predictive steps comparison for vehicle localization,” in *International Conference on Intelligent Robots and Systems*, IEEE/RSJ, 2005.
- [8] A. Kelly, “Some useful results for closed-form propagation of error in vehicle odometry,” cmu-ri-tr-00-20, Robotics Institute, Carnegie Mellon University, 2000.
- [9] X. R. Li and Z. Zhao, “Practical measures for performance evaluation of estimators and filters,” in *Workshop on Estimation, Tracking, and Fusion - A Tribute to Yaakov Bar-Shalom*, (Monterey, CA), May 2001.
- [10] R. C. Smith and P. Cheeseman, “On the representation and estimation of spatial uncertainty,” *Int. Journal of Robotic Research*, vol. 5, no. 4, winter 1986.
- [11] Y. Bar-Shalom and X. R. Li, *Estimation and Tracking: Principles, Techniques, and Software*. Boston, MA: Artech House, 1993. Reprinted by YBS Publishing, 1998.